The CYK Algorithm

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CS – 6800
Summer I - 2009

The CYK Algorithm

- *The membership problem*:
  - Problem:
    - Given a context-free grammar **G** and a string **w**
      - **G** = (V, ∑ ,P , S) where
        - » V finite set of variables
        - » ∑ (the alphabet) finite set of terminal symbols
        - » P finite set of rules
        - » S start symbol (distinguished element of V)
        - » V and ∑ are assumed to be disjoint
      - **G** is used to generate the string of a language
  - Question:
    - Is **w** in **L**(**G**)?

The CYK Algorithm

- J. Cocke
- D. Younger,
- T. Kasami

  - Independently developed an algorithm to answer this question.

The CYK Algorithm Basics

- The Structure of the rules in a Chomsky Normal Form grammar
- Uses a “dynamic programming” or “table-filling algorithm”

Chomsky Normal Form

- *Normal Form* is described by a set of conditions that each rule in the grammar must satisfy
- Context-free grammar is in CNF if each rule has one of the following forms:
  - A → BC at most 2 symbols on right side
  - A → a, or terminal symbol
  - S → λ null string

  where B, C ∈ V – {S}

Construct a Triangular Table

- Each row corresponds to one length of substrings
  - Bottom Row – Strings of length 1
  - Second from Bottom Row – Strings of length 2

    .

    .

  - Top Row – string ‘w’

Construct a Triangular Table

- $\mathbf{X_{i,\,i}}$ is the set of variables A such that A → w_i is a production of G

- Compare at most n pairs of previously computed sets:

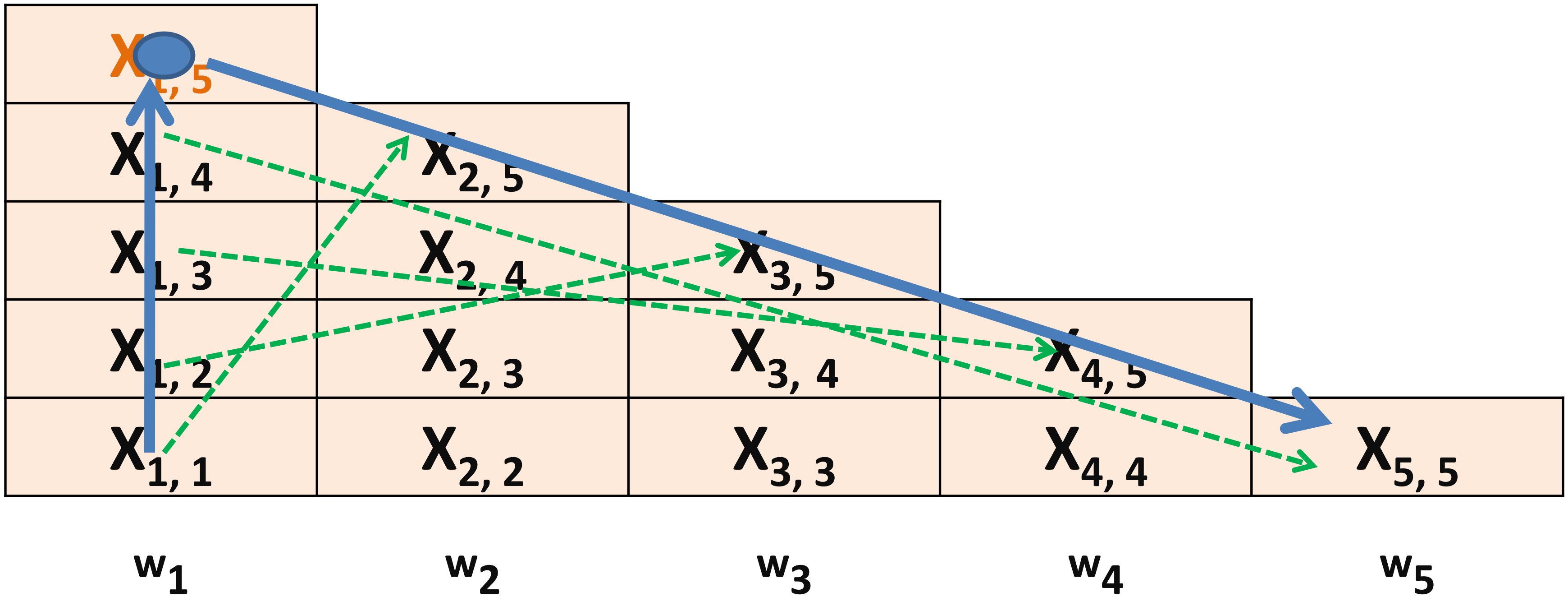
  $(\mathbf{X_{i,\,i}}, \mathbf{X_{i+1,\,j}}), (\mathbf{X_{i,\,i+1}}, \mathbf{X_{i+2,\,j}}) \ldots (\mathbf{X_{i,\,j-1}}, \mathbf{X_{j,\,j}})$

Construct a Triangular Table

$X_{1,5}$				
$X_{1,4}$	$X_{2,5}$			
$X_{1,3}$	$X_{2,4}$	$X_{3,5}$		
$X_{1,2}$	$X_{2,3}$	$X_{3,4}$	$X_{4,5}$	
$X_{1,1}$	$X_{2,2}$	$X_{3,3}$	$X_{4,4}$	$X_{5,5}$
w_1	w_2	w_3	w_4	w_5

Table for string ‘**w**’ that has length 5

Construct a Triangular Table



Looking for pairs to compare

Example CYK Algorithm

- Show the CYK Algorithm with the following example:
  - CNF grammar **G**
    - S → AB | BC
    - A → BA | a
    - B → CC | b
    - C → AB | a
  - **w** is baaba
  - Question Is **baaba** in L(G)?

Constructing The Triangular Table

S → AB | BC
A → BA | a
B → CC | b
C → AB | a

{B}	**{A, C}**	**{A, C}**	**{B}**	**{A, C}**

b **a** **a** **b** **a**

Calculating the Bottom ROW

Constructing The Triangular Table

- $\mathbf{X_{1,2} = (X_{i,i}, X_{i+1,j}) = (X_{1,1}, X_{2,2})}$
- ➔ {B}{A,C} = {BA, BC}
- Steps:
  - Look for production rules to generate BA or BC
  - There are two: S and A
  - $\mathbf{X_{1,2} = \{S, A\}}$

S → AB | BC
A → BA | a
B → CC | b
C → AB | a

Constructing The Triangular Table

{S, A}				
{B}	**{A, C}**	**{A, C}**	**{B}**	**{A, C}**
b	**a**	**a**	**b**	**a**

Constructing The Triangular Table

- $\mathbf{X_{2,3} = (X_{i,i}, X_{i+1,j}) = (X_{2,2}, X_{3,3})}$
- ➔ {A, C}{A,C} = {AA, AC, CA, CC} = Y
- Steps:
  - Look for production rules to generate Y
  - There is one: B
  - $\mathbf{X_{2,3} = \{B\}}$

S → AB | BC
A → BA | a
B → CC | b
C → AB | a

Constructing The Triangular Table

{S, A}	{B}			
{B}	{A, C}	{A, C}	{B}	{A, C}
b	a	a	b	a

Constructing The Triangular Table

- **$X_{3,4} = (X_{i,i}, X_{i+1,j}) = (X_{3,3}, X_{4,4})$**
- ➔ {A, C}{B} = {AB, CB} = Y
- Steps:
  - Look for production rules to generate Y
  - There are two: S and C
  - **$X_{3,4}$ = {S, C}**

S → AB | BC
A → BA | a
B → CC | b
C → AB | a

Constructing The Triangular Table

{S, A}	{B}	{S, C}		
{B}	{A, C}	{A, C}	{B}	{A, C}
b	a	a	b	a

Constructing The Triangular Table

- $\mathbf{X_{4,5} = (X_{i,i}, X_{i+1,j}) = (X_{4,4}, X_{5,5})}$
- ➔ {B}{A, C} = {BA, BC} = Y
- Steps:
  - Look for production rules to generate Y
  - There are two: S and A
  - $\mathbf{X_{4,5} = \{S, A\}}$

S → AB | BC
A → BA | a
B → CC | b
C → AB | a

Constructing The Triangular Table

{S, A}	{B}	{S, C}	{S, A}	
{B}	{A, C}	{A, C}	{B}	{A, C}
b	**a**	**a**	**b**	**a**

Constructing The Triangular Table

- $\mathbf{X_{1,3}} \quad \mathbf{= (X_{i,i}, X_{i+1,j})\ (X_{i,i+1}, X_{i+2,j})}$
  $\mathbf{= (X_{1,1}, X_{2,3}), (X_{1,2}, X_{3,3})}$
- ➔ {B}{B} **U** {S, A}{A, C}= {BB, SA, SC, AA, AC} = Y
- Steps:
  - Look for production rules to generate Y
  - There are NONE: S and A
  - $\mathbf{X_{1,3} = \emptyset}$
  - **no elements in this set (empty set)**

S → AB | BC
A → BA | a
B → CC | b
C → AB | a

Constructing The Triangular Table

Ø				
{S, A}	**{B}**	**{S, C}**	**{S, A}**	
{B}	**{A, C}**	**{A, C}**	**{B}**	**{A, C}**
b	**a**	**a**	**b**	**a**

Constructing The Triangular Table

- $\mathbf{X_{2,4}} \quad \mathbf{= (X_{i,i}, X_{i+1,j})\ (X_{i,i+1}, X_{i+2,j})}$

  $\mathbf{= (X_{2,2}\ , X_{3,4})\ , (X_{2,3}\ , X_{4,4})}$
- ➔ {A, C}{S, C} **U** {B}{B}= {AS, AC, CS, CC, BB} = Y
- Steps:
  - Look for production rules to generate Y
  - There is one: B
  - $\mathbf{X_{2,4} = \{B\}}$

S → AB | BC
A → BA | a
B → CC | b
C → AB | a

Constructing The Triangular Table

Ø	{B}			
{S, A}	{B}	{S, C}	{S, A}	
{B}	{A, C}	{A, C}	{B}	{A, C}
b	**a**	**a**	**b**	**a**

Constructing The Triangular Table

- $\mathbf{X_{3,5}}$ $\mathbf{= (X_{i,i}, X_{i+1,j})\ (X_{i,i+1}, X_{i+2,j})}$
  $\mathbf{= (X_{3,3}, X_{4,5}), (X_{3,4}, X_{5,5})}$
- → {A,C}{S,A} **U** {S,C}{A,C}
  = {AS, AA, CS, CA, SA, SC, CA, CC} = Y
- Steps:
  - Look for production rules to generate Y
  - There is one: B
  - $\mathbf{X_{3,5} = \{B\}}$

S → AB | BC
A → BA | a
B → CC | b
C → AB | a

Constructing The Triangular Table

Ø	{B}	{B}		
{S, A}	{B}	{S, C}	{S, A}	
{B}	{A, C}	{A, C}	{B}	{A, C}
b	**a**	**a**	**b**	**a**

Final Triangular Table

{S, A, C}	**← $X_{1, 5}$**			
Ø	**{S, A, C}**			
Ø	**{B}**	**{B}**		
{S, A}	**{B}**	**{S, C}**	**{S, A}**	
{B}	**{A, C}**	**{A, C}**	**{B}**	**{A, C}**
b	**a**	**a**	**b**	**a**

- Table for string ‘**w**’ that has length 5
- The algorithm populates the triangular table

Example (Result)

- Is baaba in L(G)?

Yes

We can see the S in the set X_{1n} where ‘n’ = 5
We can see the table
the cell X_{15} = (S, A, C) then
if S $\in$ X_{15} then baaba $\in$ L(G)

Theorem

- The CYK Algorithm correctly computes X_{ij} for all i and j; thus w is in L(G) if and only if S is in X_{1n}.
- The running time of the algorithm is $O(n^3)$.

References

- J. E. Hopcroft, R. Motwani, J. D. Ullman, *Introduction to Automata Theory, Languages and Computation*, Second Edition, Addison Wesley, 2001
- T.A. Sudkamp, An Introduction to the Theory of Computer Science Languages and Machines, Third Edition, Addison Wesley, 2006

Question

- Show the CYK Algorithm with the following example:
  - CNF grammar **G**
    - S → AB | BC
    - A → BA | a
    - B → CC | b
    - C → AB | a
  - **w** is ababa
  - Question Is **ababa** in L(G)?
- Basics of CYK Algorithm
  - The Structure of the rules in a Chomsky Normal Form grammar
  - Uses a “dynamic programming” or “table-filling algorithm”
- Complexity O(n3)